Radiant Cooling For Closed-Loop Water Containment: Exploration of Possible Application in Dry Docks

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Project Number: 507
8/20/2015

Performing Organization: SPAWAR
Sponsoring Organization: NESDI
Keywords: Dry Dock Cooling, Heat Transfer, Closed Loop, Pipe System, Cost, Pareto
List of Programs Used to Make Report: Word, Hyperlinking, and PDF

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
Aug. 2012	Final	6/15/2015-8/14/2015

4. TITLE AND SUBTITLE
Radiant Cooling for Closed-Loop Water Containment: Exploration of Possible Application in Dry Docks

5a. CONTRACT NUMBER
N/A

5b. GRANT NUMBER
N/A

5c. PROGRAM ELEMENT NUMBER
N/A

6. AUTHOR(S)
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5d. PROJECT NUMBER
N/A

5e. TASK NUMBER
N/A

5f. WORK UNIT NUMBER
N/A

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Energy & Environmental Sciences (Code 71760)
Space and Naval Warfare Systems Center Pacific

8. PERFORMING ORGANIZATION REPORT NUMBER
N/A

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Navy Environmental Sustainability Development to Integration

10. SPONSOR/MONITOR'S ACRONYM(S)
NESDI

11. SPONSOR/MONITOR'S REPORT NUMBER(S)
507

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
Models based on empirical fluid flow and heat transfer data combined with basic principle analysis provide data for estimating the cost of implementing a closed-loop radiant cooling system for ships in dry docks. Depending of the material used, pipe lengths from 300m to 3000m with diameters ranging from 10in to 24in all cooled a design load of 8 F°=4.(44)¯ K above ambient temperature when that temperature was assumed to be 300 K. From the cost analysis performed, $12 million would be the maximum initial investment for the system that moves 26 million gallons per day (≈18000gpm). This leaves $30 million to pay for the other aspects of implementing this system.

15. SUBJECT TERMS
Dry Dock Cooling, Heat Transfer, Closed Loop, Pipe System, Cost, Pareto, Puget Sound

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
Unclassified	Unclassified	Unclassified	SAR	31	Dr. Daniel Grady
					19b. TELEPHONE NUMBER *(Include area code)* (619) 553-2793

Reset

Abstract

Models based on empirical fluid flow and heat transfer data combined with basic principle analysis provide data for estimating the cost of implementing a closed-loop radiant cooling system for ships in dry docks. Depending on the material used, pipe lengths from $300\ m$ to $3000\ m$ with diameters ranging from $10\ in$ to $24\ in$ all cooled a design load of $8\ F° = 4.\overline{44}\ K$ above ambient temperature when that temperature was assumed to be $300\ K$. From the cost analysis performed, $16 million would be the maximum initial investment for the system that moves 26 million gallons per day ($\approx 18000\ gpm$). Although this analysis has not accounted for redundancy, pumps, and other one-time installation costs, other solutions for this problem have proposed budgets of $40 million, making this solution cost-competitive.

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Foreword [1]

Most Navy vessels in dry dock only partially deactivate. Many of their components continue to operate normally and require cooling water. Currently, this water is simply pumped from the harbor, through the ship's systems, and discharged. The effluent cooling water contains a higher concentration of copper and other metals and is at a higher temperature compared to ambient conditions; this is not a concern at sea, but at dock additional environmental regulations apply. These regulations will become stricter in the short to medium term, and this will require a change in current practice. A closed-loop water cooling system would eliminate the discharge of any cooling water back into the harbor and thereby satisfy regulatory requirements.

A closed-loop cooling system solves this problem by continuously recirculating the same cooling water through a dry-docked ship. Since no water is discharged into the harbor, environmental regulations are satisfied. The challenges with closed-loop cooling systems based on traditional engineering solutions like industrial-scale refrigerator or chilling are scaling the system to accommodate the volume of cooling water required (projected to be up to 18 million gallons per day for Ford-class carriers) without making the physical footprint of the system prohibitively large, and the ongoing operating costs, in both electricity and maintenance, required for such a system.

We propose to address both of these challenges through a novel reversal of commercial technology widely used in home and office buildings: radiant floor heating.

Summary

The objective of this project is to determine if radiant cooling technology is cost-competitive and feasible enough for deployment at a Navy shipyard. Selection of various diameters and flow rates determined the required lengths of pipe to cool a $8\ F°$ load and the respective pressure losses. These lengths and pressure losses specified the system costs at various flow rates for comparison. When plotted for various diameters and numbers of modules, the costs associated with making the system to satisfy a 26 million gallon per day flow depict a tradeoff between initial and recurrent costs. Twelve possible installation sites at Puget Sound Naval Shipyard can house the proposed system. Lastly, the results show that the system will cost $16 million in material costs. Although this analysis has not accounted for redundancy, pumps, and other one-time installation costs, other solutions for this problem have proposed budgets of $40 million. Basic cost estimates for the unaccounted-for work suggest that the radiant cooling solution is cost-competitive.

Introduction

The subjects addressed here include cooling, fluid dynamics, and Pareto analysis.

Cooling is a subset of the discipline of heat transfer in which energy is extracted from a system as heat. In this subset, two modes of cooling will be addressed: conduction and convection. Conduction results from physical contact of media. Convection arises from the bulk movement of a medium. A heat transfer coefficient, defined as the amount of heat flux experienced per a known temperature difference, captures the effectiveness of a convective heat transfer process. For the case of this problem, convection occurs from a fluid flow in a pipe and conduction occurs in the soil surrounding the pipe.

Despite having many analytic models, convection is mainly an empirical or numeric science. This is especially true for turbulent flow where the bulk motion of fluid particles is chaotic and involves a lot movement besides the bulk motion of the fluid. Since this application will consistently have convection from a turbulent fluid flow, empirical equations from past literature [6] will model the convective heat transfer. These relationships give a relationship between the heat transfer coefficient and known properties of the flow being analyzed. After the heat transfer coefficient is found, the heat transfer rate of the flow can be predicted within a certain error.

Fluid dynamics becomes important any time a flowing fluid is examined. As with convection, the dynamics and behavior of turbulent flow is mainly an empirical or numeric science. The information of interest here is head loss of the flow. Head loss relates to the pressure lost from a flowing flow. It can be represented by a pressure or the height of a column of the fluid since a column of fluid exerts a pressure proportional to its density and the gravitational acceleration (i.e. $p = \rho g h$). With a known head loss, a pump can be sized, or chosen, so that it can maintain the desired flow.

A Pareto chart shows options that cannot have improvement in one area without causing a decline in another. Such a chart reveals the existence of a tradeoff between components of the options being compared without a way of knowing the value of each area and how the values relate. All shown points on a Pareto chart are the most relatively efficient options in the space depicted by the axes used to plot the chart.

Notkin Engineering previously did an extensive engineering report on solutions to the dry dock problem. The report addressed many solutions that did not include this radiant cooling option. In the conclusion, adjusting current infrastructure to meet stricter regulations and still cool the ships solved the problem most cost-effectively. Improvements cost an estimated \$42 million at the least and \$49 million at the most. [3]

The purpose of this investigation is to determine the cost-competitiveness and feasibility of implementing a system that cools a fluid passively in the ground for use in dry docks. First, the resulting equations from analysis are presented with the assumptions used. Second, the process for determining the cost of the system is discussed. Third, the results of the equations are displayed graphically and cost comparisons are made. Fourth, areas in Puget Sound Naval Base that provide enough space for construction are identified. Finally, a recommendation with extra thoughts will be provided on what option to use. The scope of this exploration encompasses the cost associated with pipe and estimated power costs for the system, not the cost of the emergency redundancy, pumps, excavation, and planting of the system in the ground. From this, managers and engineers working in dry docks can determine whether this system would be a wise investment and what pumps to use later.

Methodology

Two analytic methods determined the length required for the necessary heat transfer of the problem and the resulting pressure loss from this length when bent into a radiator like form. All calculations used SI units. When English units were the desired inputs or outputs, their metric equivalents were actually calculated and then converted. Appendix A holds the detailed derivations of the equations.

For the heat transfer analysis, assumptions made computation tractable. First assumption: the fluid in the pipe flowed in a steady, fully-developed, incompressible stream. As the interest is in the performance of a turbulent flow in steady state, steady flow is a valid assumption, and the flow would reach full development quickly. The fluid will consist of a liquid, and most liquids compress negligibly. Second assumption: the flow maintains a constant average velocity since the pipe diameter is to remain constant. Third assumption: the system losses energy only by heat transfer. This is sensible because the system will have no work extracting components. Fourth assumption: all friction factors are Darcy friction factors as it is the most commonly used parameter in the derivations of the pipe flow equations. Fifth assumption: the characteristic length of the system is the pipe diameter because the equations used follow this. Lastly, the pipe will initially be highly conductive so that the conduction resistance across the pipe could be neglected.

Fig. 1 shows the simplified diagram for the heat transfer analysis.

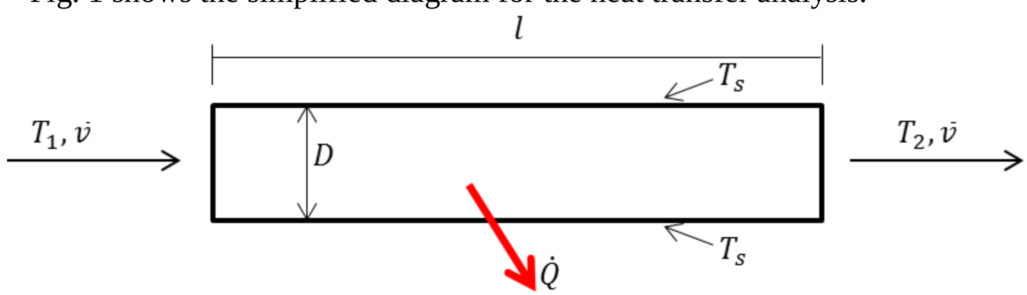

Fig. 1 – Simple Heat Transfer Model Sketch

Using this figure, a simple energy balance found the heat lost as a product of the Nusselt number, surface temperature gradient, and surface area. To evaluate the surface temperature gradient, the average bulk temperature of the fluid was used and the surface was held at constant temperature. After some algebra, the length of the pipe follows equation (1):

$$l = \frac{\rho D^2 \bar{v} c_p (T_1 - T_2)}{2(Nu\, k)(T_1 + T_2 - 2T_s)} \qquad (1)$$

where the Nusselt number is evaluated using an empirical relationship. If the pipe was not conductive, the heat lost would be represented as a function of the same product as before but with the Nusselt number term replaced by the reciprocal of the sum of the thermal resistances of the convective transfer and conductive transfer. This results in equation (2):

$$l = \frac{FA + \sqrt{(FA)^2 + 4FBCDE\pi}}{2CDE\pi} \qquad (2)$$

$$A = 2\pi D k_p;\ B = Nu\, k \ln((D + 2t)/D);\ C = 2\pi k_p Nu\, k;$$

$$E = (T_1 + T_2 - 2T_s);\ F = \frac{\rho \bar{v} \pi D^2}{2} c_p (T_1 - T_2)$$

For the pressure drop analysis, two additional assumptions are made. First new assumption: the system remains at a constant elevation in the ground. This is reasonable

since the system is expected to be installed horizontally. Second new assumption: the system will consist of a known number of 180° bends and straight sections of pipe denoted by β and γ respectively.

The pressure drop analysis started with a mechanical energy balance and simplified to the following relationship:

$$\Delta p = \rho \left(\gamma K + \beta f \frac{l}{D}\right) \frac{\bar{v}^2}{2} \tag{3}$$

With both of these relationships, various design values gave length and pressure loss values for various diameters ($10\ in - 24\ in$), pipe materials (conductive and insulative), and flow rates. Conductive materials had thermal conductivities above $15\ W/mK$. A flow rate of interest was 26 million gallons per day. The knowns of the relationships came from a working fluid of 25% sodium chloride (NaCl) aqueous solution, a bend of $R = 4.5D$ (D is the diameter of the pipe), a smooth schedule 40 pipe ($\epsilon = 0$), a cooling load of $8\ F°$ above soil temperature, a soil temperature of $300K$, and the system having 5 bends and 6 straight sections. These knowns can be updated in the Python notebook in Appendix B to regenerate the analysis for the new information.

Costs were determined from the results of the two analyses. The total initial cost of the system was calculated as the cost of the required length of pipe. The ongoing cost was kept in terms of the power usage of the system for easy conversion later. Using the lengths calculated, the cost of insulative piping came from the typical price values of PVC pipe at various diameters advertised in 2015 [4]. Conductive piping costs were found for $10\ in$ and $12\ in$ diameters [7]. The rest were estimated by taking the insulative costs and multiplying by the ratio of metal pipe cost and plastic pipe at $12\ in$ diameter. Appendix D summarizes the prices used. The power required for the flow was calculated using the pump power equation assuming a pump efficiency of $\eta = 82\%$ with the equation $P = \dot{V}\Delta P\eta$.

The above costs cover implementation of one big system to sate the needed flow rates. In practice, many smaller yet similar systems, called modules, work together to cover the desired flow rates. This increases redundancy and improves the pressure head. With the division into modules, each module will experience less total flow rate affecting the required lengths and power costs. Total costs for a modularized system are the cost of each module times the number of modules.

There already existed a tradeoff between power and material costs but now there is also a dependence on how many modules will be used. A Pareto Chart will depict the tradeoff present when evaluating costs at the flow of interest (26 gallons per day), various diameters, and various numbers of modules. Once this chart is created, various cost estimates will help in comparing costs.

To determine where the system could be implemented, the total area was estimated. With the knowns above, a rectangle could enclose the design according to equation (4):

$$A = \left(\frac{(l-\gamma 2\pi R)}{\beta} + 2R\right)(\gamma 2R) \tag{4}$$

A large length of about $1650\ m$ was used to get a rectangle that was big enough for most cases. The amount of area was located throughout the Puget Sound Naval Base in space where there was only continuous parking lot, grass, or gravel on the main land. The area was assumed to be malleable, so the original rectangle was not always maintained but the area remained essentially the same. A different color designates each separate viable location.

Results and Discussion

The heat analysis equations gave graphical outputs of Fig. 2 and 3.

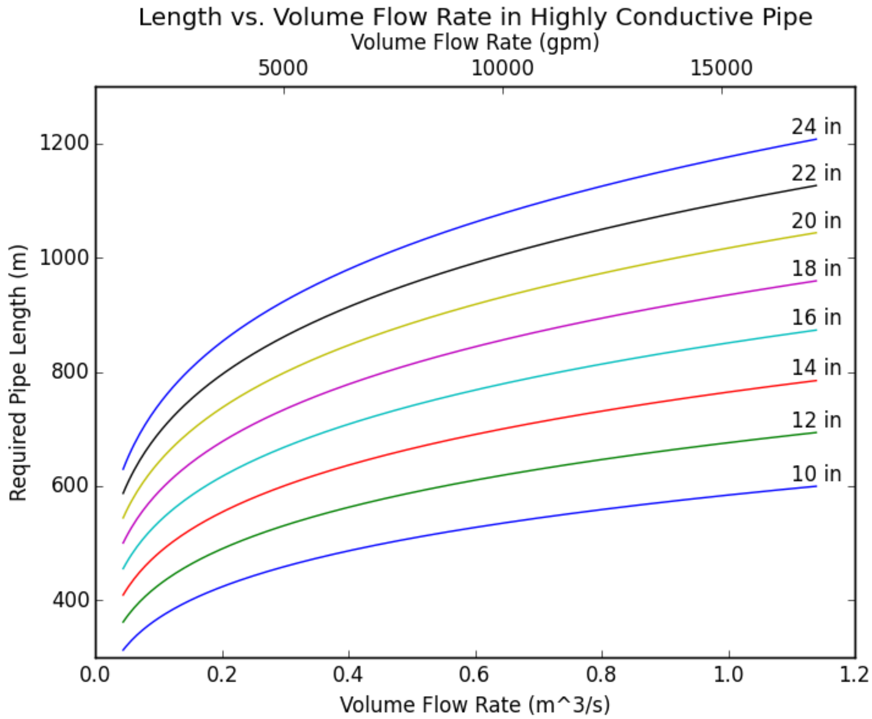

Fig. 2 – Required Length vs. Flow Rate – Conductive Material

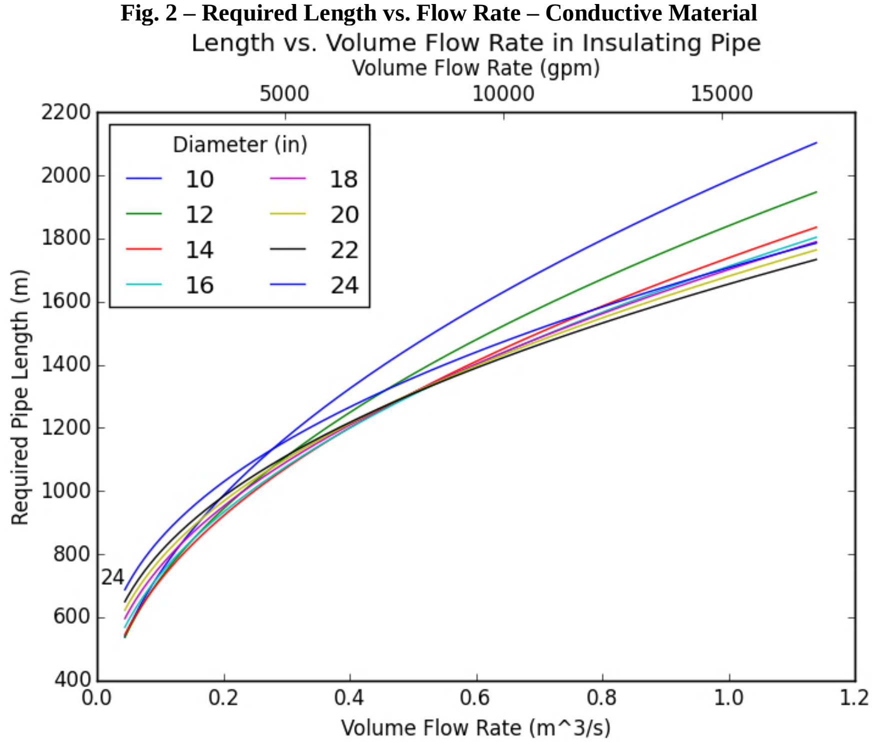

Fig. 3 – Required Length vs. Flow Rate - Insulative Material

A length range from 300 m to 2200 m contains all the observed possibilities. All curves share the same trends that length increases with flow rate and bigger pipe diameters result in longer pipes at lower flow rates.

The equations used in developing the heat transfer analysis will carry on the errors associated with the values they use. If the fluid property and geometry values can be considered as absolute constants (i.e. no error associated with them), the error in the length will equal the error, in percent, of the calculated Nusselt number. This would give an error of less than one percent for the flows observed [2]. The crossing of curves for the insulative pipes occurs due to the subtle effect of the thickness variations for different diameters of schedule 40 pipe.

The pressure analysis equations gave graphical outputs of Fig. C1 and C2 located in Appendix C. As a general trend, the pressure grows rapidly with increasing flow rates, following a square function (i.e. x^2) to be exact. Also pressure increases with decreasing diameters. With the same concessions as with the heat transfer, the pressure loss calculation gets its error from the friction factor calculation [6] and the length. This results in around a 2.25% error.

The cost analysis gave Fig. C3, C4, C5, and C6, located in Appendix C. The costs for the pipe inherit the error of 1% from the length calculation while the power cost receives the 2.25% error from the pressure values. Fig. C7, located in Appendix C was created from combining the values found above evaluated at a flow rate of 26 million gallons per day, various diameters, and various module numbers. When Fig. C7, had decidedly worse options removed, Fig. 4, a Pareto Chart, results:

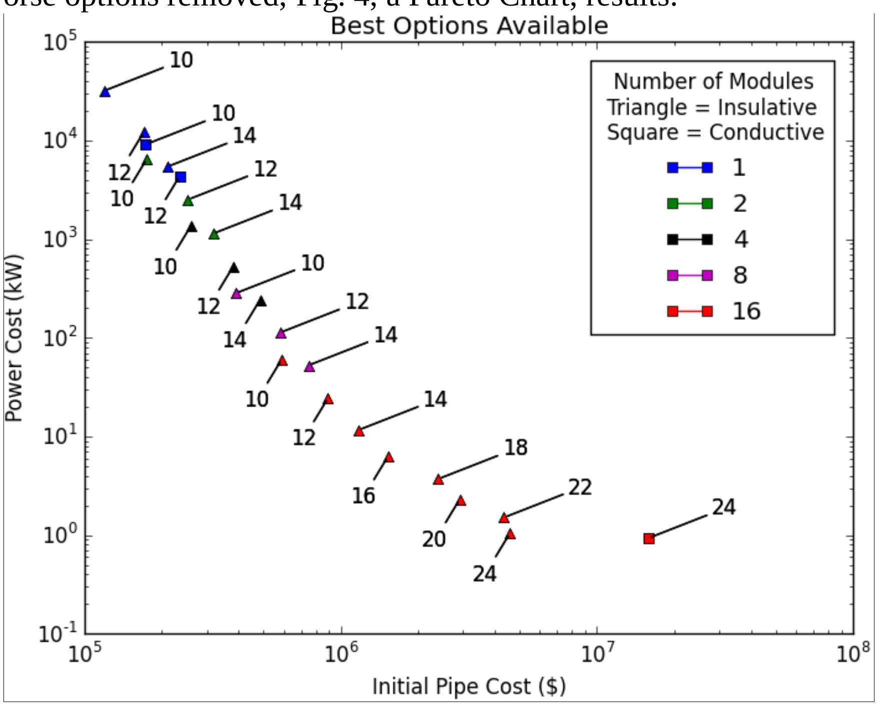

Fig. 4 – Optimal System Layouts

After noting that this graph has the errors from the different costs, the most expensive option initially is less than $16 million.

Twelve million dollars does not account for emergency redundancy, pump costs, and excavation costs. However, as a naïve estimates, pump costs could total to around $1.6 million for 16 pumps at $100,000 a pump. The cost of digging is estimated to around $14 million [5]. To estimate the cost of redundancy, 10% will be added to the total so far (i.e. $3.16 million). This projects the total of the system to cost around $35 million compared to the $42 million proposed by Notkin Engineering. The cost of a specific system is broken down in Appendix E resulting in about $27 million.

A total area of 7950 m^2 would contain the system. The 12 spots found in Puget Sound Naval Base are shown on a Google Earth image in Fig. 5.

Fig. 5 – Viable Area Locations

As Fig. 5 shows, there are many valid locations. However, all of these are not in the direct vicinity of the dry docks to be serviced and would require piping to connect the areas with the dry dock.

Conclusions

The cost of the system to cool an 8 $F°$ load in 26 gallons per day with a 25% NaCl aqueous solution in a pipe system would cost around $16 million. Although this analysis has not accounted for redundancy, pumps, and other one-time installation costs, other solutions for this problem have proposed budgets of $40 million. Basic cost estimates for the unaccounted-for work suggest that the radiant cooling solution totals around $27 million for a specific system. There exist 12 areas where this system can be implemented. Overall this system is both cost-effective and feasible.

Recommendations

This analysis has shown that a radiant cooling system is a cost-competitive option to address stricter environmental restrictions and larger cooling demands at dry docks. This is an expensive option but possibly cheaper than other proposed ones. Overall, this report has shown that this technology should at least be considered for implementation at dry docks. The cost estimates should be considered as naïve estimates. Further research or contracting can look into true costs associated with implementing this system in Puget Sound Naval Base.

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[1] Grady, Daniel. (2014). RADIANT COOLING FOR CLOSED-LOOP WATER CONTAINMENT. *NESDI Project Number 507*.

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Appendix A - In-Depth Analyses

Pipe Length Analysis for Pipe System

Description: A level pipe system (ps) is proposed as to carry water in a radiant cooling project design. As a modeling task, the length required for reaching a specified temperature in a simplified setup, i.e. along a straight pipe in the ground, is to be analyzed. The goal is to find a relationship between l, D, and heat lost.

Given:

- D - diameter of the pipe system
- $\bar{v}$ - average speed of fluid in the ps
- ρ - the density of working fluid
- μ - dynamic viscosity of working fluid
- T_1 - inlet design temperature
- T_2 - outlet design temperature
- k - thermal conductivity of working fluid
- α - thermal diffusivity of working fluid
- T_s - temperature of the soil surrounding the pipe

Determine: The length of pipe (l) required for given design temperatures

Model: 1) Steady, fully developed, incompressible flow

2) Constant average speed
3) Turbulent flow
4) Heat transfer is sole mode of energy loss
5) Darcy friction factors
6) Highly conductive pipe

Basic Equations: Reynolds Number Equation

$$Re = \frac{\rho D v}{\mu} \qquad (1)$$

Prandtl Number

$$Pr = \frac{\nu}{\alpha} \qquad (2)$$

Nusselt Number

$$Nu = \frac{hD}{k} \qquad (3)$$

Nusselt Number Empirical Equation by Gnielinsk

$$Nu = \frac{(f/8)(Re-10^3)Pr}{1+12.7(f/8)^{1/3}(Pr^{2/3}-1)} \quad \begin{cases} 3000 < Re < 10^6 \\ 0.5 < Pr \end{cases} \qquad (4)$$

Nusselt Number Empirical Equation 2

$$Nu = 0.023\, Re^{0.8} Pr^{n} \quad \begin{cases} 10^4 < Re \\ 0.5 < Pr \\ n = 0.33 \; for\ cooling^{1} \\ n = 0.4 \; for\ heating \end{cases} \qquad (5)$$

General Heat Loss Equation (similar to Newton's Law of Cooling)

$$\dot{Q} = hA\Delta T \qquad (6)$$

Steady State Conservation of Energy Equation

[1] https://en.wikipedia.org/wiki/Heat_transfer_coefficient referencing F. Kreith, ed. (2000). *The CRC Handbook of Thermal Engineering*. CRC Press.

$$\dot{Q} + \dot{W} = \dot{m}c_p\Delta T \qquad (7)$$

Analysis:

First use (1) and (2) with the given information to get Re and Pr. From this determine whether to use (4) or (5) to calculate the Nusselt Number. Using (4) requires knowledge of the Friction Factor of the flow. The Friction Factor coefficient is a nondimensional term dependent on the Reynold's number and Surface Roughness coefficient ($\frac{\epsilon}{D}$). With assumption 1, it can be found using a Moody Plot or an empirical fitting equation like the Colebrook equation:

$$\left(\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\frac{\epsilon}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)\right).$$

Now solve for the Nusselt Number for use in (3) to find the heat transfer coefficient of the process:

$$h = \frac{Nu\, k}{D} \qquad (8)$$

Next (7) is simplified with assumption 4 and (6) and (8). Equation (6) uses the temperature of the soil and the average temperature for the temperature gradient causing the heat flow out of the pipe. The use of soil temperature is justified by assumption 6:

$$\left(\frac{Nu\, k}{D}\right) A(T_{ave} - T_s) = \dot{m}c_p(T_1 - T_2) \qquad (9)$$

$$=> \left(\frac{Nu\, k}{D}\right)\pi D l(T_{ave} - T_s) = \frac{\rho\pi D^2}{4}\bar{v}c_p(T_1 - T_2)$$

$$=> (Nu\, k)l\left(\frac{T_1}{2} + \frac{T_2}{2} - T_s\right) = \frac{\rho D^2}{4}\bar{v}c_p(T_1 - T_2)$$

$$=> (Nu\, k)l(T_1 + T_2 - 2T_s) = \frac{\rho D^2}{2}\bar{v}c_p(T_1 - T_2)$$

$$=> l = \frac{\rho D^2\bar{v}c_p(T_1 - T_2)}{2(Nu\, k)(T_1 + T_2 - 2T_s)} \qquad (10)$$

Equation (10) gives the length required for the desired temperature difference for a characterized flow in a straight pipe.

Discussion:

One of the major factors that made this solvable was choosing the characteristic length of the flow to be the diameter of the pipe not the to-be-solved for length of the pipe. This analysis provides a relationship between the length, diameter, and a desired heat loss through design temperatures. The cost of computing this simple result will be in finding f and calculating Nu.

The assumptions used in this analysis are justified. The steady, fully-developed flow aligns with the desire for a steady state solution. Water is often incompressible in applications such as this. With assumption 1, constant average speed remains valid as long as the pipe diameter is constant. Heat transfer being the only form of loss is true since there are no work-extracting elements. Darcy Friction Factors were assumed in all equation calculations as it is the most commonly used parameter and the derivations of the equations[2] imply its use. Lastly, assumption 6 is valid since the hope is to have a highly conductive pipe material for this application.

If this last assumption did not hold, the left hand side of (9) would have an extra thermal resistance term. This term would be a thermal resistor added in series to the heat transfer

[2] Mills, A. (1999). *Basic Heat and Mass Transfer* (2nd ed.). Los Angeles, CA: Pentice Hall.

coefficient term (which is representing a conductance). This would change the equation in the following way:

$$\frac{Nu\,k}{D} => \frac{1}{\frac{D}{Nu\,k}+\frac{\ln((D+2t)/D)}{2\pi k_p l}}$$

where t is the thickness of the pipe and k_p is the thermal conductivity of the pipe. This makes the relationship between l, D, and the heat loss more complicated:

$$l = \frac{FA+\sqrt{(FA)^2+4FBCDE\pi}}{2CDE\pi} \qquad (11)$$

$$A = 2\pi D k_p,\ B = Nu\ k\ln((D+2t)/D),\ C = 2\pi k_p Nu\ k$$

$$E = (T_1 + T_2 - 2T_s),\ F = \frac{\rho\bar{v}\pi D^2}{2} c_p(T_1 - T_2)$$

The one major thing that is uncertain is how the soil temperature in contact with the pipe is to be modeled. The following example assumes a constant soil temperature. With all the assumptions present, the values found with this analysis are low end estimates.

<u>Example:</u>

For this project, it is expected that there will be a large flow rate of water. To estimate this number, a baseline of $22 * 10^6 gal/day$ was suggested. This in SI units is:

$$22 * 10^6 \frac{gal}{day}\left|\frac{1\ day}{24\ hr}\right|\left|\frac{1\ hr}{60\ min}\right|\left|\frac{1\ min}{60\ s}\right|\left|\frac{0.00378541178\ m^3}{1\ gal}\right| = 0.9638778\frac{m^3}{s} \approx 0.964\frac{m^3}{s}$$

Volume flow rate is related to speed and diameter by:

$$\dot{V} = Av = \frac{\pi D^2}{4}v => v = \frac{4\dot{V}}{\pi D^2}$$

Choosing a pipe diameter of $D = 0.3m$ requires $\bar{v} \approx 13.64\frac{m}{s}$.

The remaining data needed for this problem are calculated at room temperature (i.e. $300K$) for salt water (25% by mass NaCl solution[3]) since the temperature changes are not expected to be drastic enough for property changes and salt water is the possible working fluid:

$$\rho = 1178\frac{kg}{m^3},\ \mu = 1.65 * 10^{-3}\ \frac{N\cdot s}{m^2}, c_p = 3310\frac{J}{kgK}, k = 0.470\frac{W}{mK}, T_1 = 308K,$$
$$T_2 = 300K$$

With this information, the Reynold's number, thermal diffusivity, kinematic viscosity, and Prandtl number are calculated is calculated:

$$Re = \frac{\rho v D}{\mu} = \frac{1178*13.64*0.3}{1.65*10^{-3}} = 2.92144 * 10^6, \alpha = \frac{k}{\rho c_p} = \frac{0.470}{1178*3310}\frac{m^2}{s} \approx 1.2053816 * 10^{-7}\frac{m^2}{s}$$

$$\nu = \frac{\mu}{\rho} = \frac{1.65*10^{-3}}{1178}\frac{m^2}{s} \approx 1.400679 * 10^{-6}\frac{m^2}{s}, Pr = \frac{1.400679*10^{-6}}{1.2053816*10^{-7}} \approx 11.6202122215$$

use (5) in (10):

$$l = \frac{\rho D^2\bar{v}c_p(T_1-T_2)}{2(0.023\,Re^{0.8}Pr^{0.33}k)(T_1+T_2-2T_s)} = \frac{1178*0.3^2*13.64*3310(8)}{2(0.023*(2.92144*10^6)^{0.8}*(11.620212)^{0.33}*0.470)(8)} \approx 662\ m$$

[3] Mills, A. (1999). *Basic Heat and Mass Transfer* (2nd ed.). Los Angeles, CA: Pentice Hall.

Pressure Analysis for Pipe System

Description: A level pipe system (ps) in a radiator like layout is proposed as a pipe layout to carry water in a radiant cooling project design. The system will be created from a specified number of standard straight and bent sections, β and γ respectively

Given:

- D - diameter of the pipe system
- $\bar{v}$ - average speed of fluid in the ps
- l - length of straight part of system
- R - radius of curvature of the ps bend
- ρ - the density of water

Determine: The pressure loss throughout the system

Model: 1) Steady Incompressible Flow
2) No working components in the system
3) Constant average speed
4) Constant elevation
5) No heat or internal energy converts to mechanical energy

Basic Equations: Control Volume Conservation of Energy

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int e\rho dV + \int_A \left(u + \frac{v^2}{2} + gz + \frac{pV}{m}\right)\rho v \cdot dA \qquad (1)$$

Energy per unit Mass

$$e = u + \frac{v^2}{2} + gz \qquad (2)$$

Conservation of Mechanical Energy

$$\frac{\Delta p}{\rho} + \left(\alpha_2 \frac{\overline{v_2}^2}{2} - \alpha_1 \frac{\overline{v_1}^2}{2}\right) + \Delta(gz) = -h \qquad (3)$$

Head Loss in Straight Pipes

$$h_M = f\frac{l}{D}\frac{v^2}{2} \qquad (4)$$

Head Loss in Pipe Bends

$$h_m = K\frac{v^2}{2} \qquad (5)$$

Analysis: by applying the assumptions 1 and 2 to (1) and substituting in (2) one obtains (3):

$$\dot{Q} - \dot{W} = \frac{d}{dt}\int e\rho dV + \int_A \left(u + \frac{v^2}{2} + gz + \frac{pV}{m}\right)\rho v \cdot dA$$

$$\xrightarrow{1\ \&\ 2} \dot{Q} = \int_A \left(u + \frac{v^2}{2} + gz + \frac{p}{\rho}\right)\rho v \cdot dA \xrightarrow{(2)} \dot{Q} = \int_A \left(u + \frac{v^2}{2} + gz + \frac{p}{\rho}\right)\rho v \cdot dA$$

$$\dot{Q} = \dot{m}\left(u_2 - u_1 + gz_2 - gz_1 + \frac{p_2}{\rho} - \frac{p_1}{\rho}\right) + \int_A \left(\frac{{v_2}^2}{2}\right)\rho v \cdot dA - \int_A \left(\frac{{v_1}^2}{2}\right)\rho v \cdot dA$$

$$= \dot{m}\left(u_2 - u_1 + gz_2 - gz_1 + \frac{p_2}{\rho} - \frac{p_1}{\rho}\right) + \dot{m}(\alpha_2 \frac{\overline{v_2}^2}{2} - \alpha_1 \frac{\overline{v_1}^2}{2})$$

$$\xrightarrow{5} \frac{\dot{Q}}{\dot{m}} - (u_2 - u_1) = -h = \left(\frac{\Delta p}{\rho} + \alpha_2 \frac{\overline{v_2}^2}{2} - \alpha_1 \frac{\overline{v_1}^2}{2} + g\Delta z\right)$$

The derivation was shown to explain the α parameters as constants determined by the flow type that allow the use of the average velocity to calculate the required integrals of the fluid flow. This expression ultimately says that the changes in mechanical energy of a fluid flow result in head loss when there is heat flow from the system or energy

storage in the fluid. All this leads to using (3) as the base equation. Now apply assumptions 3 and 4 to simplify:

$$-h = \left(\frac{\Delta p}{\rho}\right) => \Delta p = -h\rho \qquad (6)$$

Since the interest is just in the magnitude of the pressure change the negative will be ignored from here on out.

The head loss term (h) consists of two terms: one caused by friction in the straight pipe components and one caused by losses in the bend components. Assuming that the ps is made from a combination of β straight components and γ bent components, the total head loss will just be a sum of all the component head losses:

$$h = \left(\gamma K \frac{v^2}{2} + \beta f \frac{l}{D} \frac{v^2}{2}\right) \qquad (7)$$

Here the loss coefficient K is a function of the nondimensional term $\frac{R}{D}$ and can be found in appropriate tables for 180° bends. The friction coefficient is a nondimensional term dependent on the Reynold's number ($Re = \frac{\rho v D}{\mu}$) and Surface Roughness coefficient ($\frac{\epsilon}{D}$) found using a Moody Plot or an empirical fitting equation like the Colebrook equation ($\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$). For laminar flows, there is an analytic expression of $f = \frac{64}{Re}$ which is captured with the Moody plot. Lastly, the speeds to use in the equation are the speeds at the respective positions in the pipe where the loss is occurring. With assumption 3, the speed is a constant value shared everywhere.

From these conditions, combining (6) and (7) yields

$$\Delta p = \rho \left(\gamma K + \beta f \frac{l}{D}\right) \frac{\bar{v}^2}{2} \qquad (8)$$

Discussion: This analysis provides a simple relationship between the pressure loss, length, diameter, and radius of bend curvatures for a desired design speed. The cost of computing this simple result will be in finding K and f for the different geometries and materials used. The assumptions used in this analysis are justified. The steady flow aligns with the desire for a steady state solution. Water is often incompressible in applications such as this. The no working components assumption was made so that this analysis can help size pumps to counteract the pressure drop. With assumption 1, constant average speed remains valid as long as the pipe diameter is constant. Constant elevation results from the fact that the ps is supposed to be level. Lastly, assumption 5 is valid since there is a natural tendency for mechanical work to convert to heat loss or an increase in internal energy. When the opposite does occur, the amount converted to mechanical energy is often insignificant. If the exact system is not as depicted in the figure above, the total head loss term will equal to the sum of (4) using the total length of straight pipe and terms from (5) for each different type of non-straight pipe with corresponding K values.

Example:

For this project, it is expected that there will be a large flow rate of water. To estimate this number, a baseline of $1000 gal/min$ was suggested. This in SI units is:

$$1000 \frac{gal}{min} \left|\frac{1\,min}{60\,s}\right| \left|\frac{0.00378541178\,m^3}{1\,gal}\right| = 0.06309196 \frac{m^3}{s} \approx 0.06309 \frac{m^3}{s}$$

Volume flow rate is related to speed and diameter by:

$$\dot{V} = Av = \frac{\pi D^2}{4} v \Rightarrow v = \frac{4\dot{V}}{\pi D^2}$$

Choosing a pipe diameter of $D = 0.3m$ requires $\bar{v} \approx 0.8925 \frac{m}{s}$.

The length of the Thompson Drydock is about $259\ m$[4]. Assuming that the project would not span an entire drydock length, the straight length will be set to $l = 100\ m$. Then, for convenience, $R = 4.5D = 1.35m$. There will be 5 bends and 6 straight segments. With the five bends having the given radius, this ps still fits into the width of the referenced Thompson Drydock. Last of the given data would be the fluid properties of water: $\rho = 1000 \frac{kg}{m^3}$ & $\mu = 8.93 * 10^{-4}\ \frac{N \cdot s}{m^2}$.

With this information, the Reynold's number is calculated:

$$Re = \frac{\rho v D}{\mu} = \frac{1000 * 0.892 * 0.3}{8.93 * 10^{-4}} \approx 3.00 * 10^5$$

Since the diameter of the pipe is quite large compared to the possible roughness lengths of the pipe, the pipe can be considered smooth (i.e. $\frac{\epsilon}{D} \approx 0$). With this knowledge, the Moody chart gives:

$$f = 0.0145$$

Using the loss coefficient chart[5], $K \approx 0.3$ for this example (thanks to the choice of $\frac{R}{D} = 4.5$).

The rest is evaluation using (8):

$$\Delta p \approx 1000 \left(5 * 0.3 + 6 * 0.0145 \frac{100}{0.3}\right) \frac{0.8925^2}{2} Pa \approx 12.15\ kPa$$

[4] http://www.oracleireland.com/Ireland/Countys/antrim/Shipyard/thompson-dock.htm

[5] http://www.thermopedia.com/content/577/?tid=104&sn=1422

Appendix B - Python Notebook Documentation

```
In [ ]: # Import needed packages as abbreviations
        import matplotlib.pyplot as plt
        import numpy as np

In [ ]: #CHANGING the values will mess up some of the labeling formats
        #Define important values of the fluid bulk (assumed to be same as surface)
        rho = 1178;                                         #Density of fluid (kg/m^3)
        mu = 1.65*10**(-3);                                 #Dynamic viscosity of fluid (N*s/m^2)
        cp = 3310;                                          #Specific heat of fluid (J/kg/K)
        k = 0.470;                                          #Thermal conductivity of fluid (W/m/K)
        T1 = 300+8*5./9.;                                   #Inlet temperature designed to be 8 deg
                                                            #    F above ambient (K)
        T2 = 300.0001;                                      #Outlet temperature (K)
        D_eng = np.arange(10,25,2);                         #Diameters in inches for fomatting
        D = D_eng*0.0254;                                   #Diameters of pipe converted from
                                                            #    inches to meters

        #Calculate the length of D
        Dlength = len(D);

        e = 0;                                              #Characteristic Roughness length
        Ts = 300;                                           #Temperature of surrounding soil
        kp = 0.04;                                          #Thermal Conductivity of the pipe (W/m/K)

        #Pipe thickness of schedule 40 pipe converted from inches to meters
        #      10   12  14   16  18   20   22   24          (in) diameter pipe
                                                                #corresponding to the thickness
        t = [0.37,0.41,0.44,0.5,0.56,0.59,0.59,0.69]*np.full(Dlength,0.0254);

        #Set important values for pressure drop
        kCoef = 0.3                                         #Head loss coefficient
        lPipe = 239                                         #Length of parking lot (meters)
        bends = 5                                           #Number of 180 degree bends in system
        strts = 6                                           #Number of straight parts
        rBend = 4.5*D                                       #Radius of the bends (meters)

        #Set information about pricing
        #               10    12     14    16     18     20    22    24    (in) diameter pipe
                                                                #corresponding to the thickness
        insulPrice = [17.26,26.67,35.06,44.19,66.75,78.13, 83.37,83.37
                              ]/np.full(Dlength,0.3048);    #£ per meter converted from
                                                            #    per foot estimated
        condPrice = insulPrice*103.34/26.67                 #£ per meter based on how one price
                                                            #    compared to plastic of same
                                                            #    diameter
        condPrice[0] = 88.2/0.3048                          #Adjust the cost of the first item
                                                            #    to what has be quoted
        costPwr = 0.12                                      #£ per kWh
        pump_eff = 0.82                                     #Efficiency of the pump purposely
                                                            #    chosen poor

        #Calculate the thermal diffusivity and kinematic viscosity
        a = k/rho/cp                                        #(m^2/s)
        nu = mu/rho                                         #(m^2/s)

        #Define vector of various flow rates desired
        VdotG = np.arange(1,26.05,0.1)                       #Million Gallons per day

        #Convert the flow into SI units
        Vdot = VdotG*10**6*0.00378541178/24/3600             #Meters cubed per second

        #Convert to gpm
        VdotGPM = VdotG*10**6/24/60

        #Record the lengths of volume flow vectors for easy use later
        length = len(Vdot);

In [3]: matplotlib inline

In [4]: #Plot with respect to volume flow rates
        #Create figures with different handles
```

```
fig1 = plt.figure(1);                                   #For conductive length
fig2 = plt.figure(2);                                   #For insulative len
fig3 = plt.figure(3);                                   #For Pressure drop in conductive
fig4 = plt.figure(4);                                   #For Pressure drop in insulative
fig5 = plt.figure(5);                                   #For Initial Cost conductive
fig6 = plt.figure(6);                                   #For Initial Cost insulative
fig7 = plt.figure(7);                                   #For Power cost conductive
fig8 = plt.figure(8);                                   #For Power cost insulative

#Create axes for all of these
ax1 = fig1.add_subplot(111);
ax2 = fig2.add_subplot(111);
ax3 = fig3.add_subplot(111);
ax4 = fig4.add_subplot(111);
ax5 = fig5.add_subplot(111);
ax6 = fig6.add_subplot(111);
ax7 = fig7.add_subplot(111);
ax8 = fig8.add_subplot(111);

#Create twin axes to display both metric and english units side by side
ax11 = ax1.twiny();
ax22 = ax2.twiny();
ax33 = ax3.twiny();
ax44 = ax4.twiny();
ax55 = ax5.twiny();
ax55 = ax5.twiny();
ax66 = ax6.twiny();
ax77 = ax7.twiny();
ax88 = ax8.twiny();

#Create a variable for consistent formatting later
#The when variable tells what power cost graphs to graph by setting the number of
#    initial initial vector values not to be graphed. Since vectors are set from
#    small to large diameter, the smaller diameter pipes are taken out first.
when = 0;
#The whenStop variable records the last vector position that the pressure graphs
#    are shown to. If anything other than -1 it is safest to take of the formatting
#    of the second x axis designated with [*****]
whenStop = -1;

#Looping for plotting
for num in xrange(0,Dlength):

    #Create the velocities for the graphs (m/s) from flow rate and diameter
    vels = 4*Vdot/np.pi/D[num]**2;

    #Create Reynolds numbers for the flows
    Re = rho*D[num]*vels/mu;

    #Create the Prandtl Number for the flows
    Pr = nu/a;

    #Check if Pr is above 0.5, if this is false the equations used are invalid
    print Pr > 0.5

    #Preallocate an empty vector of Nusselt Numbers
    Nu = np.empty(length);

    #Calculate the nusselt numbers
    for dum in xrange(0,length):

        #Determine which way to calculate the Nusselt Number
        if Re[dum] < 3000:
            Nu[dum] = 3.66;
        elif Re[dum] < 10**4: #this used to be 10**6 but is now just set to when
                              #    the other equation is not applicable
            #Approximate Friction factor adds 2% error
            fr = (-1.8*np.log10((e/D[num]/3.7)**1.11+6.9/Re[dum]))**(-2);
            Nu[dum] = ((Re[dum]-10**3)*Pr*fr/8)/(1+12.7*(Pr**(2/3)-1)*(fr/8)**(1/3));
        else:
            Nu[dum] = 0.023*(Re[dum]**0.8)*(Pr**0.33)

    #Calculate the length 1 required to get the temperature drop
    l1 = np.divide(((rho*D[num]**2*vels*cp)*(T1-T2)),((2*Nu*k*(T1+T2-2*Ts))));

    #Calculate length 2 using chunks
```

```
A = 2*np.pi*D[num]*kp;
B = np.log((D[num]+2*t[num])/D[num])*Nu*k;
C = Nu*k*2*np.pi*kp;
E = T1+T2-2*Ts;
F = (rho*D[num]**2*vels*cp)*(T1-T2)/2*np.pi;
l2 = (F*A+np.sqrt((F*A)**2+4*E*C*np.pi*D[num]*F*B))/(2*E*C*np.pi*D[num]);

#Preallocate an empty vector of friction factors for pressure calculation
f = np.empty(length);

#Calculate the friction factors
for dum in xrange(0,length):

    #Determine which way to calculate the friction factor
    if Re[dum] < 3000:
        f[dum] = 64/Re[dum];
    else:
        #Approximate Friction factor adds 2% error
        f[dum] = (-1.8*np.log10((e/D[num]/3.7)**1.11+6.9/Re[dum]))**(-2);

#Calculate the pressure drops in bar //feet if the comments are taken away
pLoss1 = rho*(bends*kCoef+f*(l1-bends*np.pi*2*rBend[num])/D[num])*vels**2/2/10**5\
         #*10**5/rho/9.8/0.3048; #What to uncomment starts if gone
pLoss2 = rho*(bends*kCoef+f*(l2-bends*np.pi*2*rBend[num])/D[num])*vels**2/2/10**5\
         #*10**5/rho/9.8/0.3048; #What to uncomment starts if gone

#Calculate the cost for the different materials by calculating the power required
pcostDay1 = pLoss1*10**2*Vdot/pump_eff;  #units kPa*m^3/s
pcostDay2 = pLoss2*10**2*Vdot/pump_eff;  #units kPa*m^3/s

#Plot the l1 data
ax1.plot(Vdot,l1)
#Annotate along the way
pt = -1
ax1.annotate("%d in" %D_eng[num], xy=(Vdot[pt], l1[pt]+10),ha='center')
plt.hold

#Plot the l2 data
ax2.plot(Vdot,l2,figure=fig2)
#Label the graphs with certain diameters
if num == Dlength-1:
    pt = 0
    ax2.annotate("%d" %D_eng[num], xy=(Vdot[pt], l2[pt]+20),ha='right')
plt.hold

#Plot the pressure data
if num >= when:
    ax3.plot(Vdot[0:whenStop],pLoss1[0:whenStop],figure=fig3)
    plt.hold
    ax4.plot(Vdot[0:whenStop],pLoss2[0:whenStop],figure=fig4)
    plt.hold

#Annotate the pressure data
if num == 0:
    pt = 120
    ax3.annotate("%d in" %D_eng[num], xy=(Vdot[pt], pLoss1[pt]),ha='right')
    ax4.annotate("%d in" %D_eng[num], xy=(Vdot[pt], pLoss2[pt]+10),ha='right')
plt.hold

#Plot the initial cost of stuff - conductive
intCost1 = condPrice[num]*l1;
ax5.plot(Vdot,intCost1);
#Annotate along the way
pt = -1
ax5.annotate("%d in" %D_eng[num], xy=(Vdot[pt], intCost1[pt]+6000),ha='center')
plt.hold

#Plot the initial cost of stuff - insulative
intCost2 = insulPrice[num]*l2;
ax6.plot(Vdot,intCost2);
#Annotate along the way
if num == 0:
```

```
        pt = 0
        ax6.annotate("%d" %D_eng[num], xy=(Vdot[pt], intCost2[pt]-3000),ha='right')
    plt.hold

    if num >= when:
        #Plot the power cost of conductive
        ax7.plot(Vdot[0:whenStop],pcostDay1[0:whenStop]);
        plt.hold
        if num == 0:
            pt = 120
            ax7.annotate("%d in" %D_eng[num], xy=(Vdot[pt], pcostDay1[pt]+200),
                         ha='right')
            ax8.annotate("%d in" %D_eng[num], xy=(Vdot[pt], pcostDay2[pt]+200),
                         ha='right')

        #Plot the power cost of insulative
        ax8.plot(Vdot[0:whenStop],pcostDay2[0:whenStop]);
        plt.hold

#Labels for the graphs
#Create number of columns to make lengend not over lap data
ncol = 2

#[*****] Label the always the same x axes
for num in xrange(0,8):
    funstr = "ax%d.set_xlabel('Volume Flow Rate (m^3/s)')" %(num+1)
    funstr2 = "ax%d%d.set_xlabel('Volume Flow Rate (gpm)')" %((num+1),(num+1))
    funstr3 = "ax%d%d.set_xlim(VdotGPM[0],VdotGPM[-1])" %((num+1),(num+1))
    eval(funstr)
    eval(funstr2)
    eval(funstr3)


#Conductive Pipe Required Length Figure
ax1.set_ylabel('Required Pipe Length (m)')
ax1.set_title('Length vs. Volume Flow Rate in Highly Conductive Pipe\n\n')

#Insulative Pipe Required Length Figure
ax2.set_ylabel('Required Pipe Length (m)')
ax2.set_title('Length vs. Volume Flow Rate in Insulating Pipe\n\n')
ax2.legend(D_eng[np.arange(when,Dlength,1)], loc = 0, ncol=ncol,
           title='Diameter (in)')

#Conductive Pipe Pressure Loss Figure
ax3.set_ylabel('Pressure Loss (bar)')
ax3.set_title('Pressure Loss in Conductive Pipes\n\n')
ax3.legend(D_eng[np.arange(when,Dlength,1)], loc = 0, ncol=ncol,
           title='Diameter (in)')

#Insulative Pipe Pressure Loss Figure
ax4.set_ylabel('Pressure Loss (bar)')
ax4.set_title('Pressure Loss in Insulative Pipes\n\n')
ax4.legend(D_eng[np.arange(when,Dlength,1)], loc = 0, ncol=ncol,
           title='Diameter (in)')

#Conductive Pipe Initial Costs Figure
ax5.set_ylabel('Cost ($)')
ax5.set_title('Pipe Cost - Conductive\n\n')

#Insulative Pipe Initial Costs Figure
ax6.set_ylabel('Cost ($)')
ax6.set_title('Pipe Cost - Insulative\n\n')

#Conductive Pipe Power Cost Figure
ax7.set_ylabel('Power (kW)')
ax7.set_title('Power Cost - Conductive\n\n')
ax7.legend(D_eng[np.arange(when,Dlength,1)], loc = 0,title='Diameter (in)',
           ncol=ncol)

#Insulative Pipe Power Cost Figure
ax8.set_xlabel('Volume Flow Rate (m^3/s)')
ax8.set_ylabel('Power (kW)')
ax8.set_title('Power Cost - Insulative\n\n')
ax8.legend(D_eng[np.arange(when,Dlength,1)], loc = 0,title='Diameter (in)',
           ncol=ncol)
```

```
#show the plots
plt.show()
```

In [14]:
```
#Plot with power cost and initial cost with respect to each other

#Create the design variable for the total flow rate to be supplied
totVdot = 26*10**6*0.00378541178/24/3600            #Twenty-six million gpd
                                                    #    converted to m^3/s


#Create figures with different handles
fig9 = plt.figure(9);                               #For Power cost vs initial
                                                    #    cost conductive

#Create axes for all of these
ax9 = fig9.add_subplot(111);

#Create Color Code
color = ['b','g','k','m','r','c']

#Establish the array to create the number of modules
m = 2**np.arange(0,5)

#Initialization step for the vectors
pcost1 = np.empty(Dlength);
pcost2 = np.empty(Dlength);
intCost1 = np.empty(Dlength);
intCost2 = np.empty(Dlength);

#Looping for plotting conductive
for dum in xrange(0,len(m)):
    for num in xrange(0,Dlength): #Loop through the different diameters

        #Calculate the velocity for the individual modulesgraphs (m/s) from
                                                    #flow rate and diameter
        vels = 4*totVdot/np.pi/D[num]**2/m[dum];

        #Create Reynolds numbers for the flows
        Re = rho*D[num]*vels/mu;

        #Create the Prandtl Number for the flows
        Pr = nu/a;

        #Calculate the Nusselt Numbers
        Nu = 0.023*(Re**0.8)*(Pr**0.33)

        #Calculate the length 1 required to get the temperature drop
        l1 = np.divide(((rho*D[num]**2*vels*cp)*(T1-T2)),((2*Nu*k*(T1+T2-2*Ts))));

        #Calculate the friction factors
        f = (-1.8*np.log10((e/D[num]/3.7)**1.11+6.9/Re))**(-2);

        #Calculate the pressure drops in bar //feet if the comments are taken away
        pLoss1 = rho*(bends*kCoef+f*(l1-bends*np.pi*2*rBend[num])/D[num])*vels**2/2/10**5\
                 #*10**5/rho/9.8/0.3048; #What to uncomment starts if gone

        #Calculate the cost for the different materials by calculating the power required
        pcost1[num] = pLoss1*10**2*totVdot/pump_eff;  #units kPa*m^3/s

        #Calculate the cost of the pipe in the required length
        #Conductive
        intCost1[num] = condPrice[num]*l1*m[dum];

    #Plot the points
    ax9.loglog(intCost1,pcost1, '%s-s' %color[dum]);

#Looping for plotting insulative
for dum in xrange(0,len(m)):
    for num in xrange(0,Dlength): #Loop through the different diameters

        #Calculate the velocity for the individual modulesgraphs (m/s) from flow rate and
        #    diameter
        vels = 4*totVdot/np.pi/D[num]**2/m[dum];

        #Create Reynolds numbers for the flows
        Re = rho*D[num]*vels/mu;
```

```
            #Create the Prandtl Number for the flows
            Pr = nu/a;

            #Calculate the Nusselt Numbers
            Nu = 0.023*(Re**0.8)*(Pr**0.33)
            #Calculate length 2 (i.e. for insulative pipes) using chunks
            A = 2*np.pi*D[num]*kp;
            B = np.log((D[num]+2*t[num])/D[num])*Nu*k;
            C = Nu*k*2*np.pi*kp;
            E = T1+T2-2*Ts;
            F = (rho*D[num]**2*vels*cp)*(T1-T2)/2*np.pi;
            l2 = (F*A+np.sqrt((F*A)**2+4*E*C*np.pi*D[num]*F*B))/(2*E*C*np.pi*D[num]);

            #Calculate the friction factors
            f = (-1.8*np.log10((e/D[num]/3.7)**1.11+6.9/Re))**(-2);

            #Calculate the pressure drops in bar //feet if the comments are taken away
            pLoss2 = rho*(bends*kCoef+f*(l2-bends*np.pi*2*rBend[num])/D[num])*vels**2/2/10**5
                     #*10**5/rho/9.8/0.3048; #What to uncomment starts if gone
            #Calculate the cost for the different materials by calculating the power required
            pcost2[num] = pLoss2*10**2*totVdot/pump_eff;  #units kPa*m^3/s

            #Calculate the cost of the pipe in the required length
            #Insulative
            intCost2[num] = insulPrice[num]*l2*m[dum];

        #Plot the points
        ax9.loglog(intCost2,pcost2, '%s-^' %color[dum]);

    #Labels for the graphs
    #Label the always-the-same axes
    ax9.set_xlabel('Initial Cost ($)')
    ax9.set_ylabel('Power Cost (kW)')

    #Show the legend
    ax9.legend(m,loc=0,title="  Number of Modules\n Triangle = Insulative\
               \n Square = Conductive")#,ncol=ncol)

    #Add title
    ax9.set_title('Pipe Cost Comparison')

    #Label a arrow to show direction of increasing D
    ax9.annotate("Increasing D", xy=(10**6.5,10**2), xytext=(10**5.7,10**3.7),
                 arrowprops=dict(arrowstyle='->'),ha='center')

    plt.show()
```

```
In [6]: #Create A plot without purely bad options
        #Plot with respect to power cost and initial cost
        #Create the design variable for the total flow rate to be supplied
        totVdot = 20*10**6*0.00378541178/24/3600           #Twenty million gpd converted to m^3/s


        #Create figures with different handles
        fig10 = plt.figure(10);                            #For Power cost vs initial cost
                                                           #    conductive

        #Create axes for all of these
        ax10 = fig10.add_subplot(111);

        #Create Color Code
        color = ['b','g','k','m','r','c']

        #Establish the array to create the number of modules
        m = 2**np.arange(0,5)
        base = np.arange(0,5)

        #Create a 5 by len(m)*Dlength vector where first column is initial pipe cost, second
        #    column is power cost, third is a material identifier (0=conductive,
        #    1=insulative), fourth is diameter of the pipe used, and fifth is the number of
        #    modules used (m)
        #Initialization step for the vector
```

```
ptlist = np.empty([5,len(m)*Dlength*2]);
#Create a counter that starts at 0
counter = -1;

#Looping to fill the vector with points
for dum in xrange(0,len(m)):
    for num in xrange(0,Dlength): #Loop through the different diameters
        #Calculate the velocity for the individual modulesgraphs (m/s) from flow rate
        #    and diameter
        vels = 4*totVdot/np.pi/D[num]**2/m[dum];

        #Create Reynolds numbers for the flows
        Re = rho*D[num]*vels/mu;

        #Create the Prandtl Number for the flows
        Pr = nu/a;

        #Calculate the Nusselt Numbers
        Nu = 0.023*(Re**0.8)*(Pr**0.33)

        #Calculate the length 1 required to get the temperature drop
        l1 = np.divide(((rho*D[num]**2*vels*cp)*(T1-T2)),((2*Nu*k*(T1+T2-2*Ts))));

        #Calculate length 2 (i.e. for insulative pipes) using chunks
        A = 2*np.pi*D[num]*kp;
        B = np.log((D[num]+2*t[num])/D[num])*Nu*k;
        C = Nu*k*2*np.pi*kp;
        E = T1+T2-2*Ts;
        F = (rho*D[num]**2*vels*cp)*(T1-T2)/2*np.pi;
        l2 = (F*A+np.sqrt((F*A)**2+4*E*C*np.pi*D[num]*F*B))/(2*E*C*np.pi*D[num]);

        #Calculate the friction factors
        f = (-1.8*np.log10((e/D[num]/3.7)**1.11+6.9/Re))**(-2);

        #Calculate the pressure drops in bar //feet if the comments are taken away
        pLoss1 = rho*(bends*kCoef+f*(l1-bends*np.pi*2*rBend[num])/D[num])*vels**2/2/10**5\
                 #*10**5/rho/9.8/0.3048; #What to uncomment starts if gone
        pLoss2 = rho*(bends*kCoef+f*(l2-bends*np.pi*2*rBend[num])/D[num])*vels**2/2/10**5\
                 #*10**5/rho/9.8/0.3048; #What to uncomment starts if gone

        #Increment the counter
        counter = counter+1;
        #print counter

        #Calculate the costs for conductive
        ptlist[0,counter] = condPrice[num]*l1*m[dum];
        ptlist[1,counter] = pLoss1*10**2*totVdot/pump_eff;  #units kPa*m^3/s
        ptlist[2,counter] = 0;
        ptlist[3,counter] = D_eng[num];
        ptlist[4,counter] = m[dum];

        #Increment the counter
        counter = counter+1;

        #Calculate the costs for insulative
        ptlist[0,counter] = insulPrice[num]*l2*m[dum];
        ptlist[1,counter] = pLoss2*10**2*totVdot/pump_eff; #units kPa*m^3/s
        ptlist[2,counter] = 1;
        ptlist[3,counter] = D_eng[num];
        ptlist[4,counter] = m[dum];



#Get rid of for sure bad options, those that are to the top right of other points
#Create a temp variable
temp = ptlist;
print np.shape(ptlist)
for num in xrange(0,len(m)*Dlength*2):
    for dum in xrange(0,len(m)*Dlength*2):
        if temp[0,num]>=ptlist[0,dum] and temp[1,num]>=ptlist[1,dum]:
            #Make the point equal to the smaller point if it is in the top right
            #    relative to that point
            temp[:,num]=ptlist[:,dum];
```

```
#Find the unique points in temp and their indices to label the graph appropriately
nothing, indeces = np.unique(temp[0,:],return_index=True)

#Plot the remaining points with annotations
#Create an evenness counter
even = 0;
for num in indeces:
    even = even + 1;
    #Choose the symbol to be used
    if temp[2,num]==0:
        symbolUsed = '%ss' %(color[np.select(temp[4,num]==m, base)]);

    else:
        symbolUsed = '%s^' %(color[np.select(temp[4,num]==m, base)]);

    #Plot the point
    ax10.loglog(temp[0,num],temp[1,num],symbolUsed);
    #Annotate the point with diameter
    if even%2 == 1:
        ax10.annotate("%d" %temp[3,num], xy=(temp[0,num],temp[1,num]),
                      xytext=(temp[0,num]*2,temp[1,num]*1.75),
                      arrowprops=dict(arrowstyle='-'),ha='center')
    else:
        ax10.annotate("%d" %temp[3,num], xy=(temp[0,num],temp[1,num]),
                      xytext=(temp[0,num]/1.25,temp[1,num]/3),
                      arrowprops=dict(arrowstyle='-'),ha='center')

#The Legend is the same as before just I do not know how to get it to show
#Label the graph
ax10.set_title("Best Options Available")
ax10.set_xlabel('Initial Pipe Cost ($)')
ax10.set_ylabel('Power Cost (kW)')
```

Appendix C - Relevant but Less Important Curves Generated

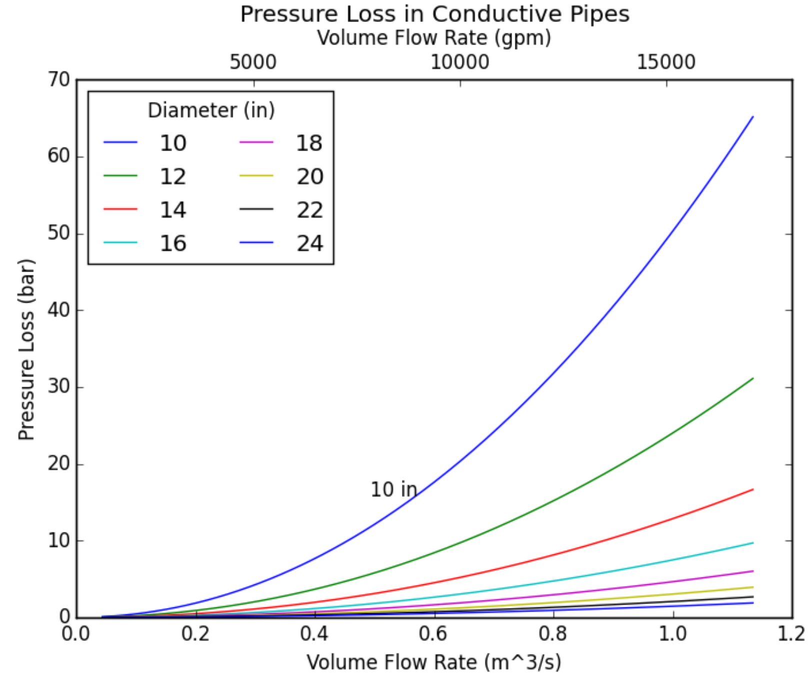

Fig. C1 – Pressure Loss in Conductive Pipes

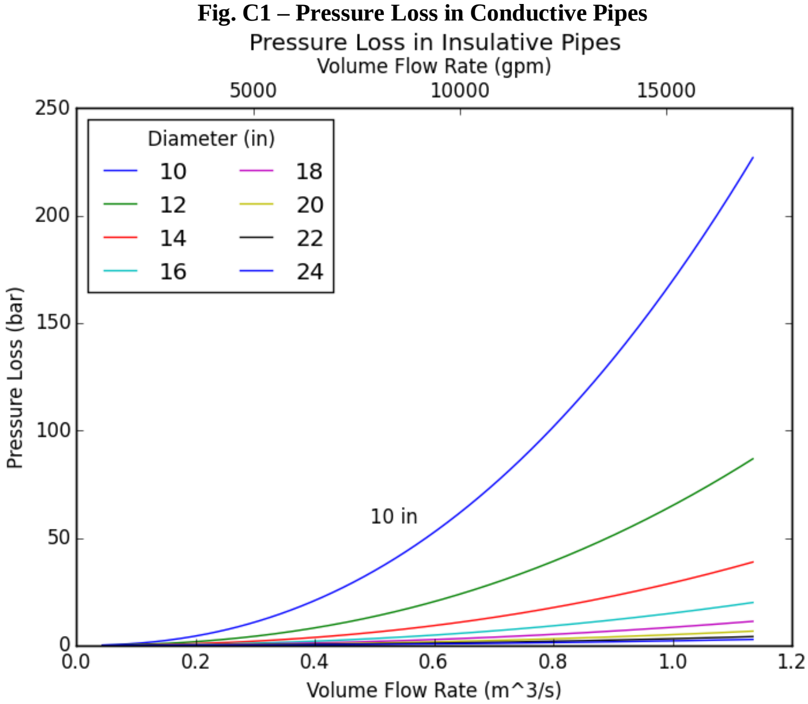

Fig. C2 – Pressure Loss in Insulative Pipes

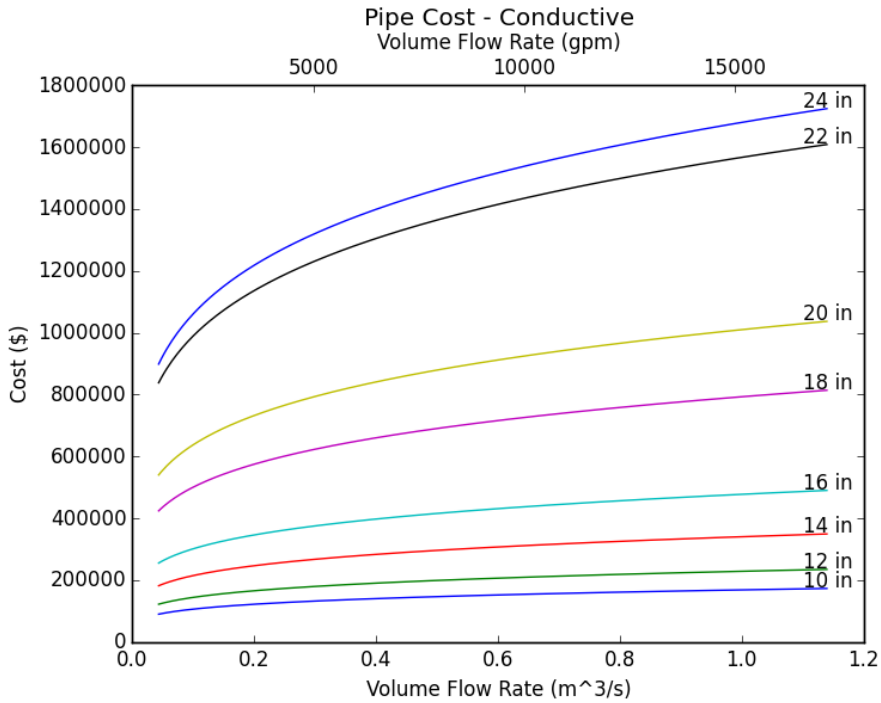

Fig. C3 – Cost of Conductive Pipe

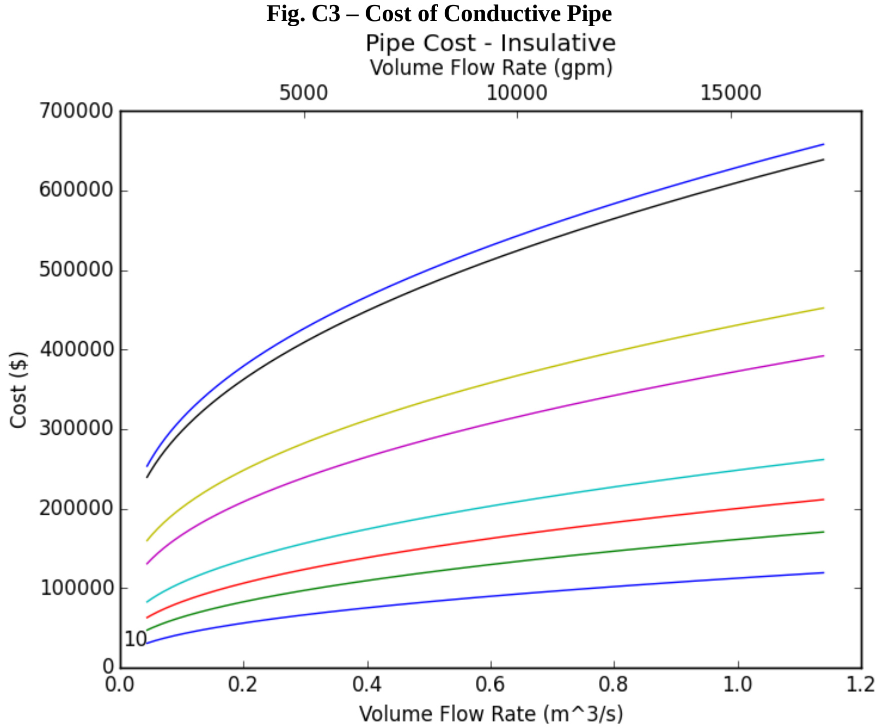

Fig. C4 – Cost of Insulative Pipe

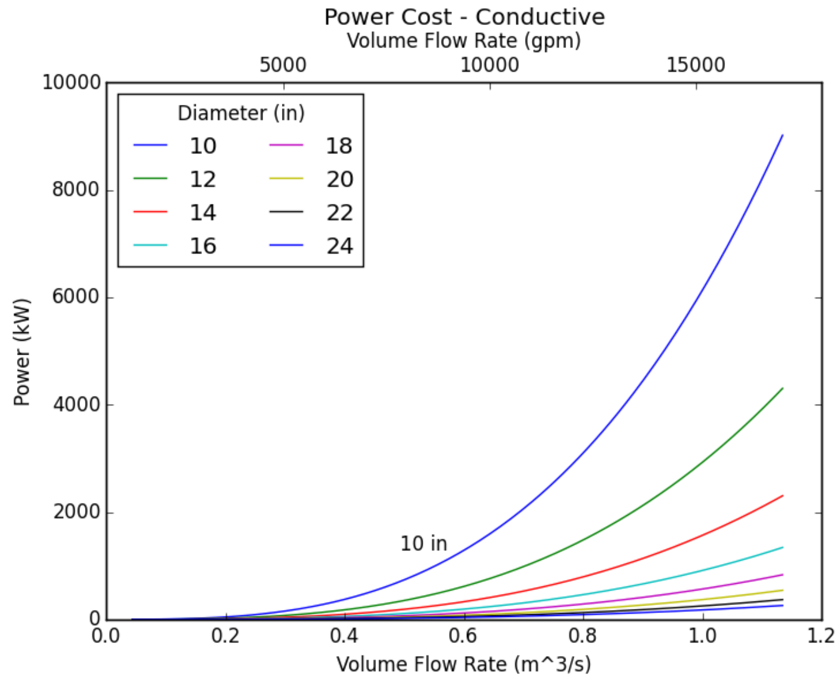

Fig. C5 – Power Cost for Conductive Pipes

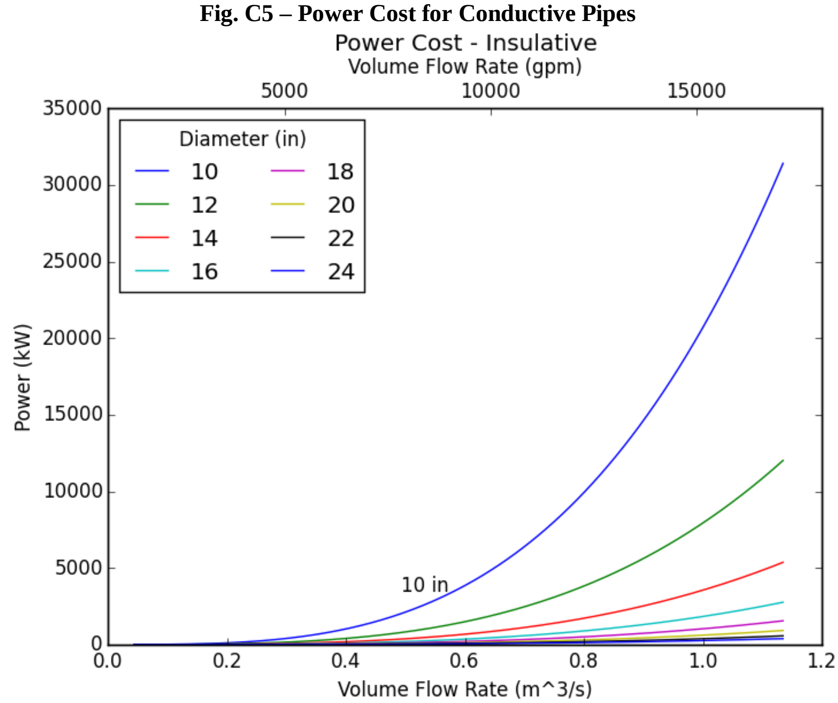

Fig. C6 – Power Cost for Insulative Pipes

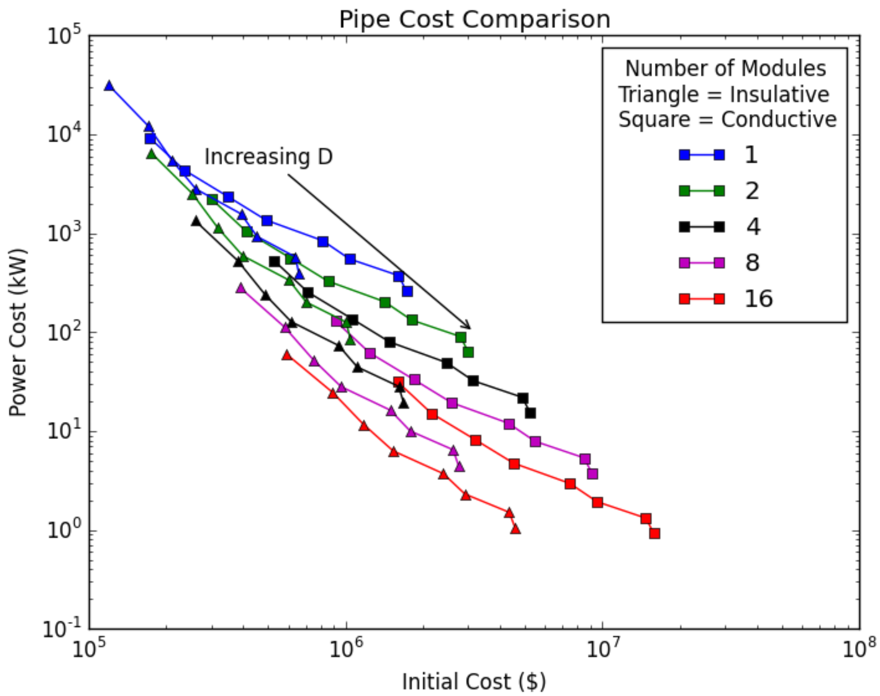

Fig. C7 – Cost Decision Chart

Appendix D - Price Summary

Diameters (in)	Insulative Pipe Cost* ($/ft)	Conductive Pipe Cost** ($/ft)
10	17.26	88.20
12	26.67	103.34
14	35.06	135.85
16	44.19	171.23
18	66.75	258.64
20	78.13	302.74
24	112.31	435.17

Notes

* - Prices based on PVC Piping as of July 2015

** - Non-hyperlinked prices based on ratio b/w the 12in diameter prices

Appendix E - Estimated Install Costs for One System Size

Pipe 24" diameter x 2276 Linear ft	$15.85	Million
Pumps 1.3 HP 1200 GPM (16 at $3,300 each)	$0.05	Million[1]
Excavation of 60000 cubic yrds	$8.68	Million[2]
10% Redundancy Costs	$2.46	Million
	$27.04	Million

1 - Price Quoted from eBay : Molds & Plastic Machinery, Inc. (2015). Aurora Pump - Centrifugal Pump, Type 411 BF, 1200 GPM, 185 Head Feet 1750 RPM. Retrieved August 6, 2015.

2 - Price Quoted from [5]

Symbol List

A	- area (for system and pipe system cross section)
D	- diameter of the pipe system (ps)
K	- pressure loss coefficient in $180°$ bends
P	- power
$\dot{Q}$	- heat transfer rate
R	- radius of curvature of the ps bend
Re	- Reynold's number
T_{ave}	- average fluid temperature
T_1	- inlet design temperature
T_2	- outlet design temperature
T_s	- temperature of the soil surrounding the pipe
$\dot{V}$	- volume flow rate
$\dot{W}$	- work rate
e	- system energy per unit mass
f	- Darcy friction factor
g	- acceleration of gravity
h	- total head loss (energy per unit mass)
h_M	- major head losses
h_m	- minor head losses
l	- length of straight part of system
k	- thermal conductivity of working fluid
k_p	- thermal conductivity of an insulative pipe
$\dot{m}$	- mass flow rate
v	- fluid speed in the ps
$\bar{v}$	- average speed of fluid in the ps
t	- insulative pipe thickness
z	- elevation relative to a referance
α	- thermal diffusivity of working fluid
β	- number of bends in the ps
Δp	- pressure drop
η	- pump efficiency
ϵ	- surface roughness
γ	- number of straight sections in the ps
μ	- dynamic viscosity of working fluid
ν	- kinematic viscosity
ρ	- the density of working fluid